GFDL's Finite-Volume Dynamical Core on the Cubed-Sphere

S.-J. Lin

Outline:

- What is a (finite-volume) "dynamical core" ?
- Fundamentals of the FV core
  - The multi-dimensional algorithm (Lin and Rood 1996)
  - Shallow water formulation (Lin and Rood 1997)
  - Finite-Volume algorithm for computing pressure gradient (Lin 1997)
  - The vertically Lagrangian finite-volume discretization
  - The non-hydrostatic solvers (RIM and SIM)
- Why do we choose the cubed-sphere grid ?
- The "island preserving" terrain filter
- Nonlinear test cases
- Two variable resolution options (stretched and nested grids)

Governing equations of a hydrostatic finite-volume model

Mass conservation:

$$\frac{\partial}{\partial t}\delta p^* + \nabla_h \cdot \left[\vec{V}\delta p^*\right] = 0$$

$$\frac{\partial}{\partial t}\left(q\delta p^*\right) + \nabla_h \cdot \left[\vec{V}q\delta p^*\right] = \boxed{S}$$

The 4 dotted red boxes (S, Fu, Fv, and H) represent “Physics” and/ or the “Chemistry”; the rest are “dynamical core”

Momentum equations:

$$\frac{\partial}{\partial t}u - \Omega v + \frac{\partial}{\partial x}\left(\frac{u^2 + v^2}{2}\right) = \frac{\delta\Phi}{\delta\pi^*}\left[\frac{\partial\pi^*}{\partial x}\right]_z + \boxed{F_u}$$

$$\frac{\partial}{\partial t}v + \Omega u + \frac{\partial}{\partial y}\left(\frac{u^2 + v^2}{2}\right) = \frac{\delta\Phi}{\delta\pi^*}\left[\frac{\partial\pi^*}{\partial y}\right]_z + \boxed{F_v}$$

1st law of thermodynamics:

$$\frac{\partial}{\partial t}\left(\Theta\delta p^*\right) + \nabla_h \cdot \left[\vec{V}\Theta\delta p^*\right] = \boxed{H}$$

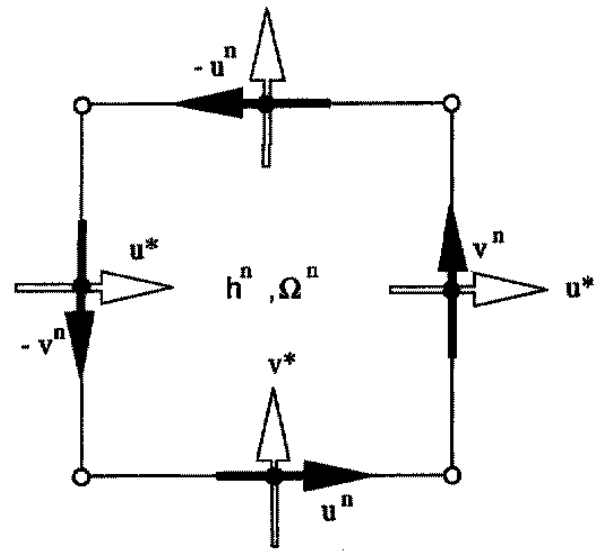
Hydrostatic balance

$$\delta\Phi = -C_p\Theta\delta\pi^*$$

Finite Volume algorithms are built on 3 basic integral theorems

- **Divergence theorem**: for constructing the flux-form advection operator

$$\iint_S \left(\nabla\cdot \rho q\vec{V}\right) ds = \oint_L \rho q\vec{V}\cdot \vec{n} dl$$

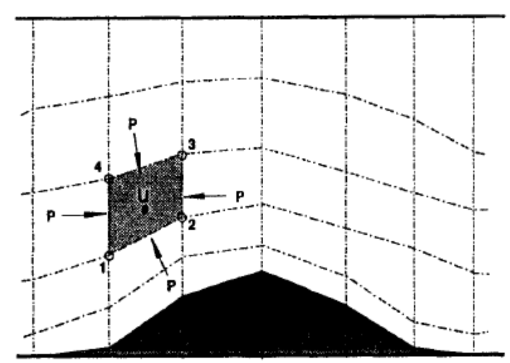


- **Stokes theorem**: for vorticity computation

$$\iint_S \left(\nabla \times \vec{V}\right)\cdot d\vec{s} = \oint_L \left(\vec{V}\cdot \vec{\sigma}\right) dl$$

- **Green's theorem**: for computing pressure gradient

$$\iint_S \frac{\partial P}{\partial x} ds = \oint_L P dx$$



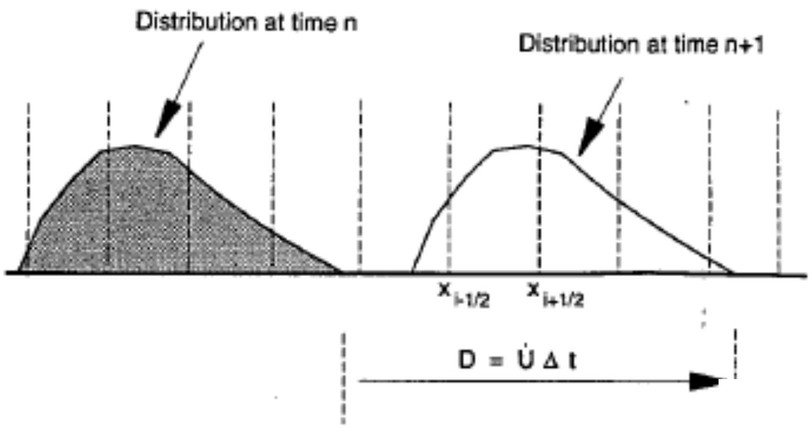
(Lin 1997, QJ)

Minimal reading list for the FV algorithms:

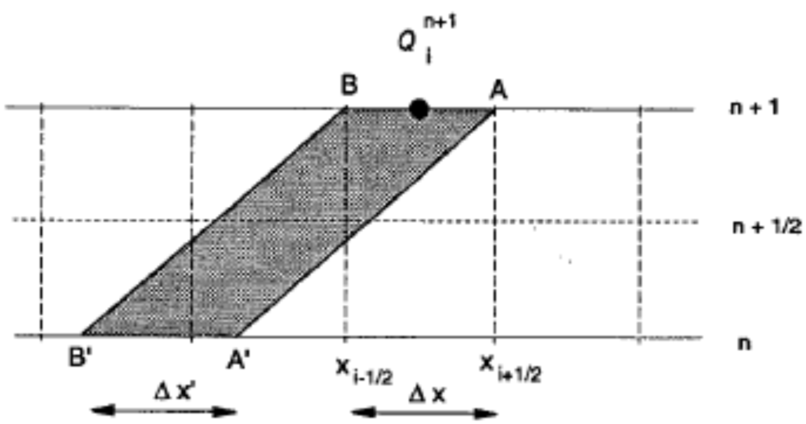
1. Advection (transport) process: *Lin and Rood 1996, Mon. Wea. Rev.*
2. Shallow water dynamical framework: *Lin and Rood 1997, QJ.*
3. Pressure gradient in general vertical coordinate: *Lin 1997, QJ.*
4. Vertical discretization: *Lin 2004, Mon. Wea. Rev.*
5. Cubed-sphere geometry: *Putman and Lin 2007, JCP*.
6. Two-way regional-global nesting: *Harris and Lin 2012, Mon. Wea. Rev.*
7. **Non-hydrostatic solvers*: Lin (manuscript; hopefully 2012)*

Physically based advection scheme

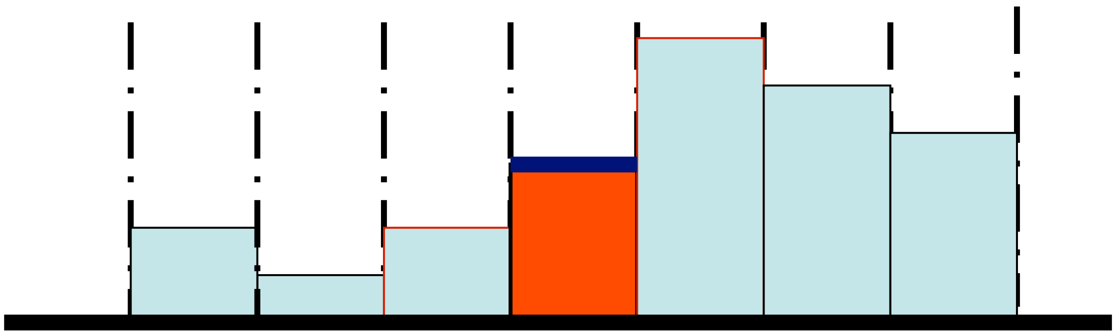
(Lin & Rood 1996, MWR)



1D transport by "shifting" the finite-volume mean grid structure (resolved + subgrid)

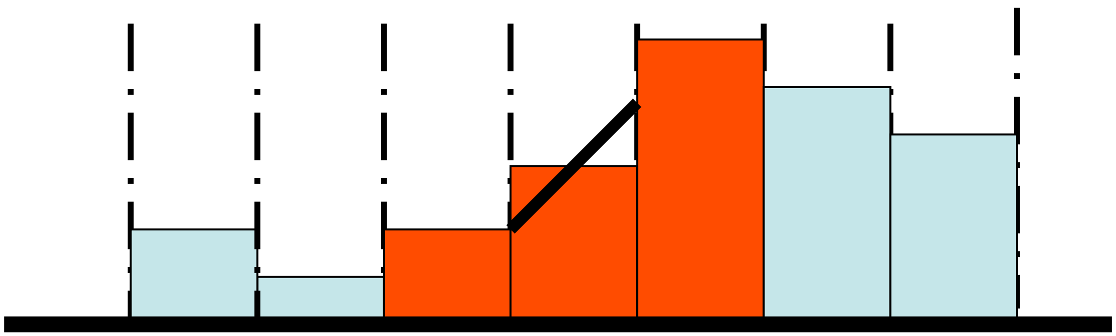


Piece-wise constant distributions are constructed using only one finite-volume mean



classical upwind scheme: 1st order accurate

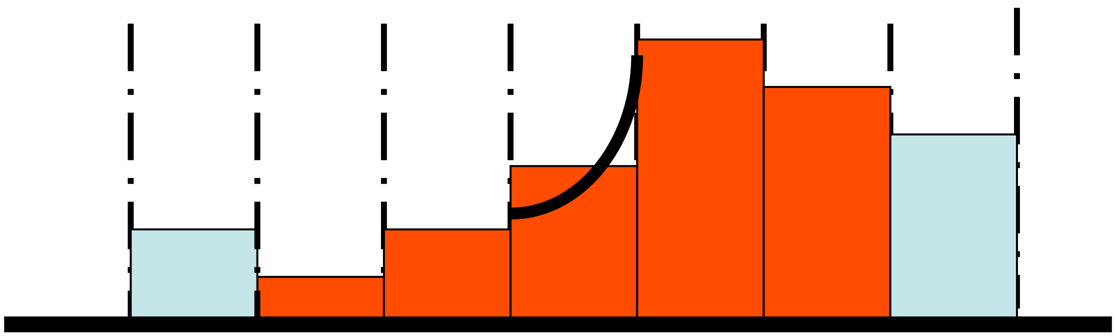
Piece-wise linear distributions are constructed using 3 finite volumes



van Leer-type schemes: 2nd order accurate

(Lin et al, 1994, MWR)

Piece-wise parabolic subgrid distributions can be constructed using only 5 finite volumes



Piecewise Parabolic method (PPM): 3rd or 4th order accurate

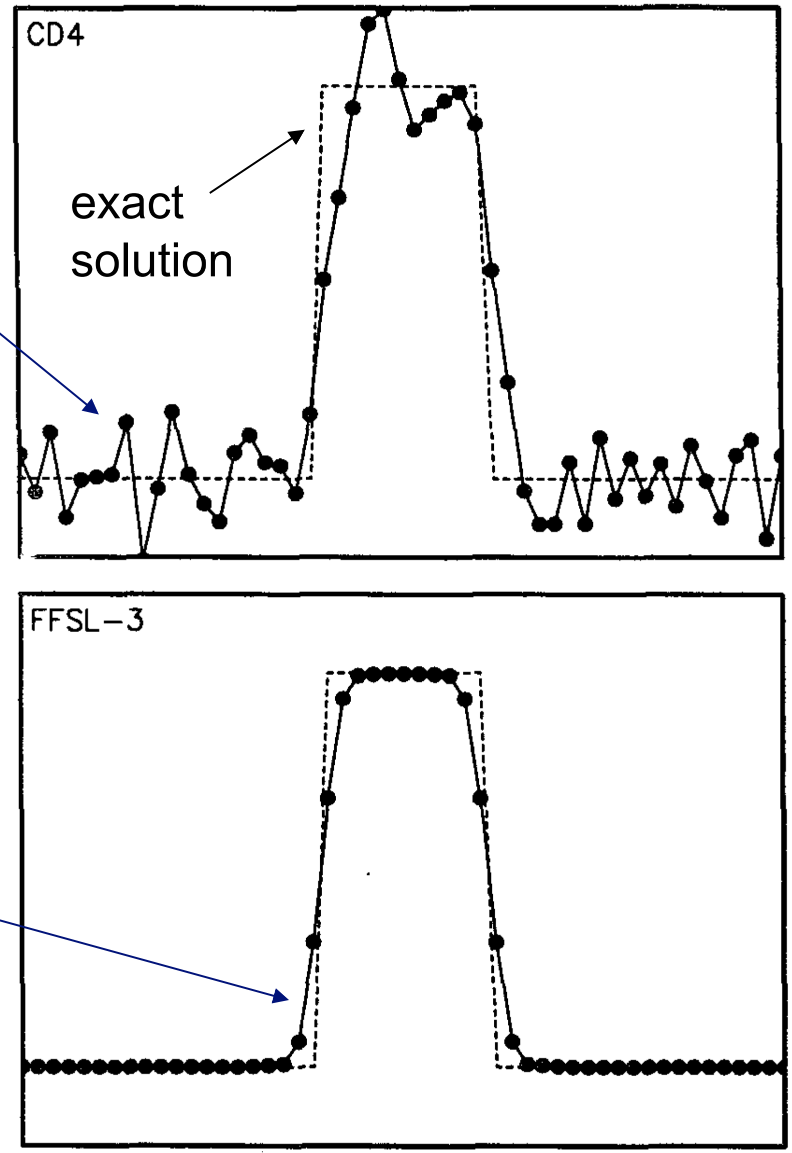
(Colella and Woodward 1984, JCP; Lin and Rood 1996, MWR)

1D Advection: mathematically vs. physically based schemes

(Lin and Rood 1996, MWR)

4th order center difference

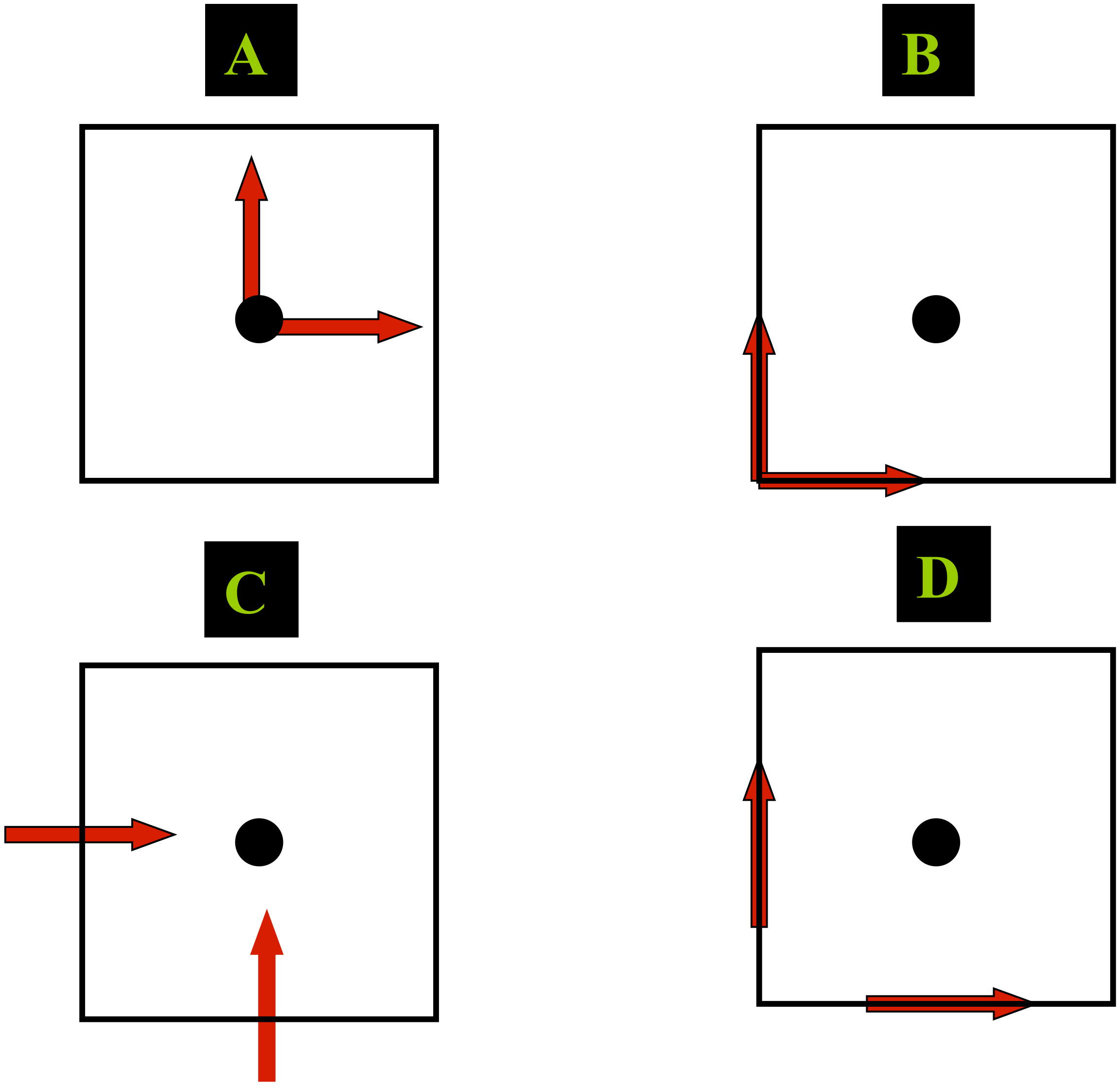
(mathematically based)



Monotonic PPM

(physically based)

The “ABC” (and D) of wind-vector staggering



Issues to consider:

- pressure gradient
- divergence
- coriolis force
- (absolute) vorticity advection (nonlinearity)
- coupling to physics & ocean

A freshman's guide on Grid staggering:
0: best 1: average 2: worst
(assuming 2nd order center difference)

Grid type	A	B	C	D
Pressure Grad	1	1	0	2
Divergence	1	1	0	2
Coriolis force	0	0	2	0 (or 2)
vorticity	1	1	2	0
Time step size*	1	2	2	1
Phys-dynamics coupling	0	2	1	1
Equal weighting sum	4	7	7	6 (or 8)

Note: higher order scheme can be used for critical operations

More notes on grid staggering:

- Based solely on conventional linear shallow-water-wave analysis (e.g., Messinger and Arakawa 1977, GARP monograh), it has been declared by some that "*C grid is the best*". However, the (vector wind) staggering is neither important nor relevant if a higher-order spatial discretization or a Riemann-solver approach is used (in the latter case one shall not stagger the prognostic variables)

- Under the <u>assumption</u> of 2nd order center differencing (as in conventional analysis), D grid is indeed the worst grid for divergence and pressure gradient computation. But most people do not realize that C grid is the worst grid for Potential Vorticity (PV) advection and Coriolis force computation and it requires the smallest time step; whereas D grid is the best choice if the PV advection approach (Lin and Rood 1997) can be adopted.

- In choosing grid staggering, non-linear behavior (in particular, vorticity/PV advection) has often been overlooked!

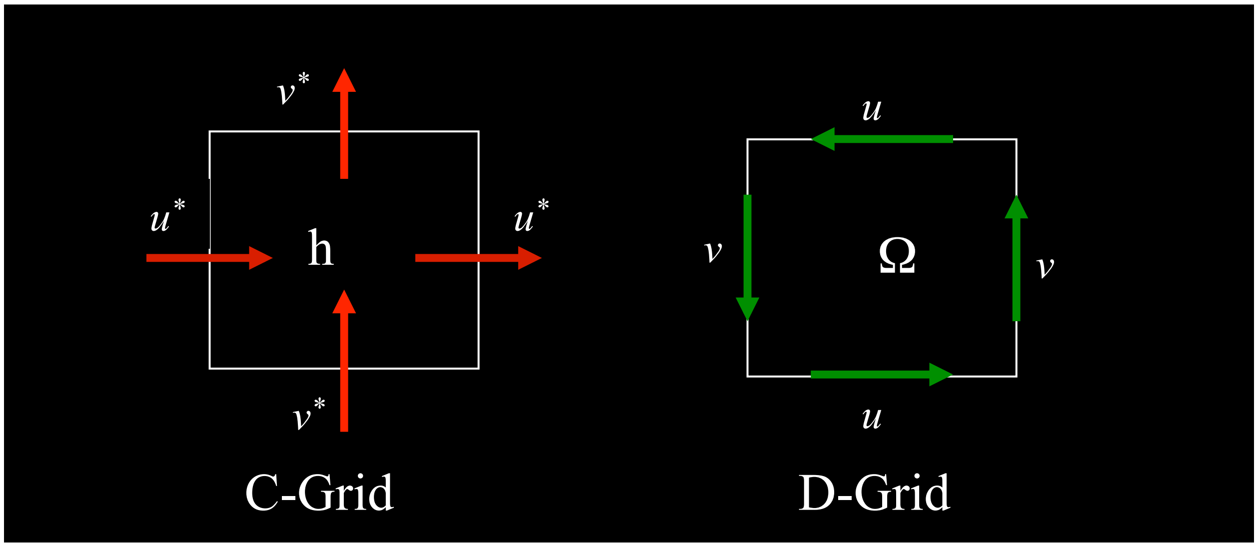
- The grid staggering issue gets more complicated in the more general (non-orthogonal) curvilinear coordinate system

- What is a good compromise between C and D grids? ***Answer***: try to combine their strength while avoiding their weakness

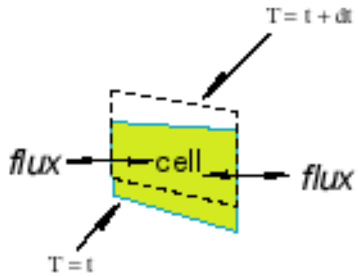
Horizontal discretization

The two-grid Approach (Lin and Rood 1997, QJ)

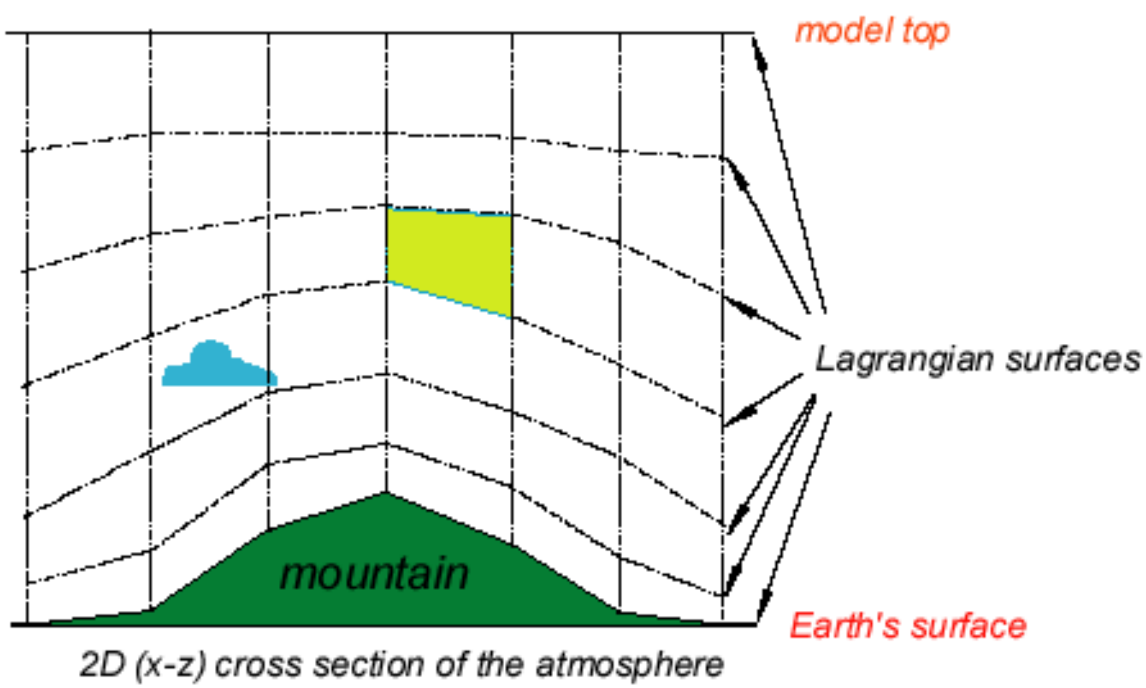
- The two-grid approach is designed to transport mass and absolute vorticity consistently (therefore, PV). It also avoids the generation of two-grid length gravity waves.
- The time-centered winds (u^*,v^*) are integrated on the C-Grid for half-time-step
- The time-centered winds are then used for the transport of the vorticity and all scalars on the D-Grid



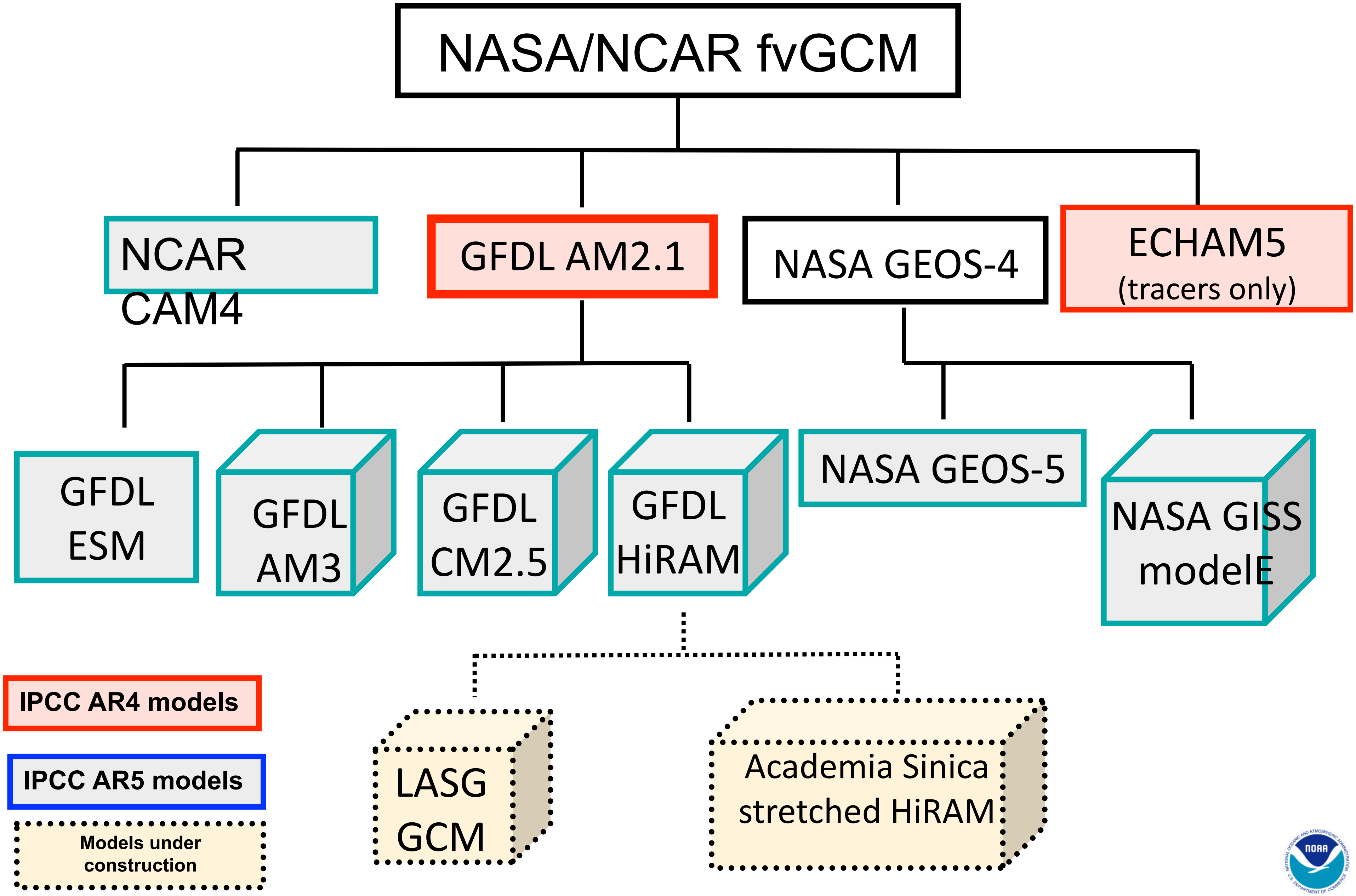
Vertically Lagrangian Control-Volume Discretization



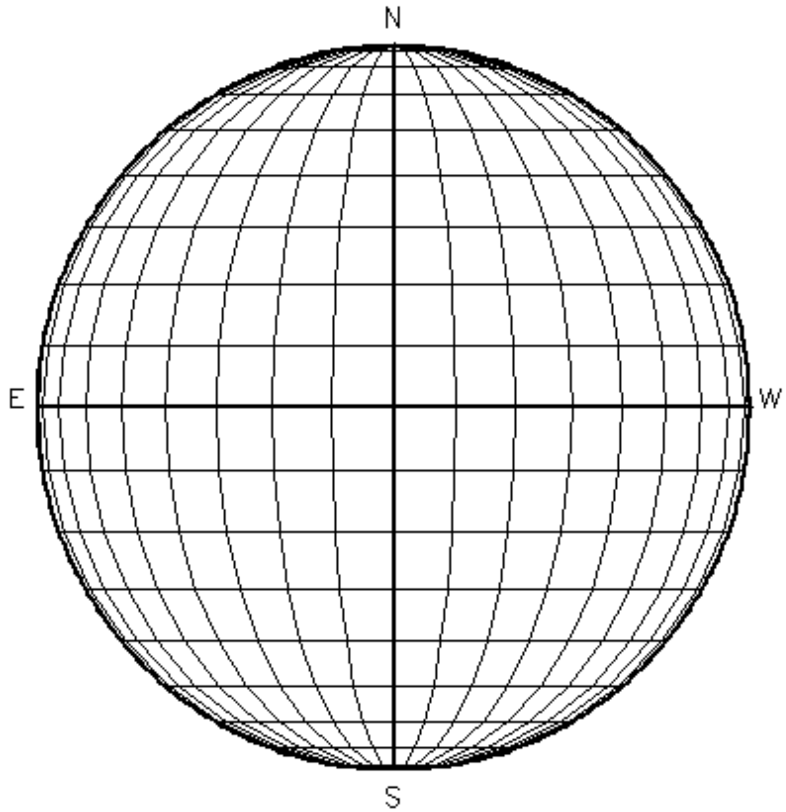
- •Horizontally Eulerian (Lin and Rood 1996)
- •Vertically Lagrangian control-volume discretization (Lin 2004)



The finite-volume model family tree



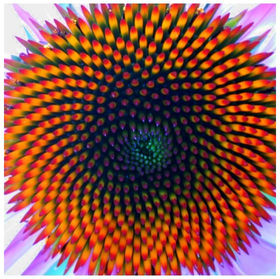
The traditional latitude-longitude grid is not suitable for ultra-high resolution modeling



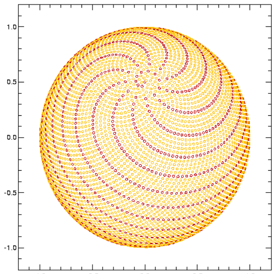
- The existence of two polar singularities prevented effective 2D domain decomposition
- The extreme grid aspect ratio at poles poses a scientific difficulty for non-hydrostatic dynamical core formulation

The search for the optimal grid on the sphere

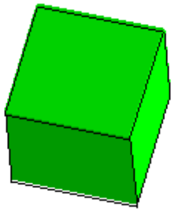
coneflower



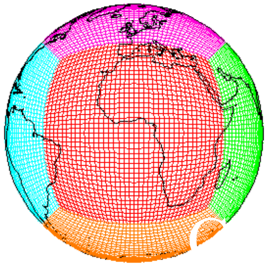
Fibonacci grid



Hexahedron/cube

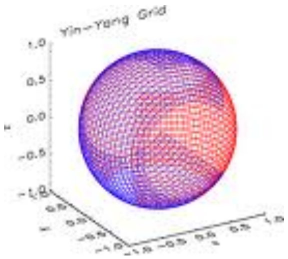


Cubed-sphere grid

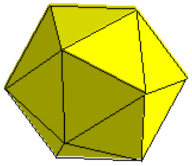


Yin-Yang

Yin-Yang grid

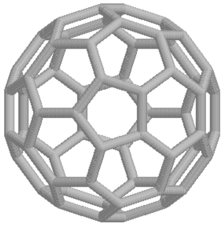


Icosahedron (Plato, ~ 400 BC)

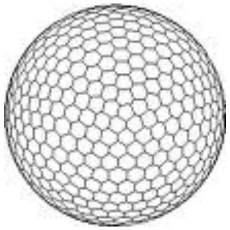


C60

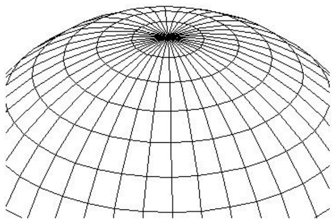
(1996 Nobel chemistry price)



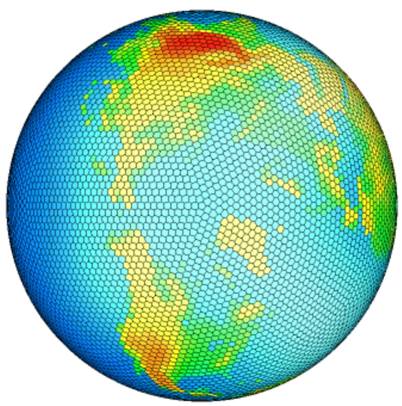
Geodesic grid



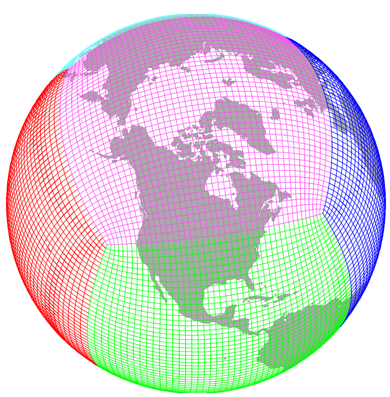
Formulating the geophysical flow equations on the sphere



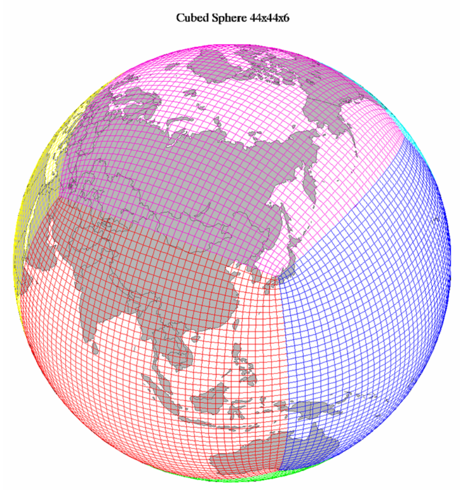
Spherical coordinate: orthogonal, extreme grid aspect ratio with two singularities; not suitable for high-resolution



Icosahedral grid: non-orthogonal, most uniform grid aspect ratio with no true singularity (but with 12 pentagons)

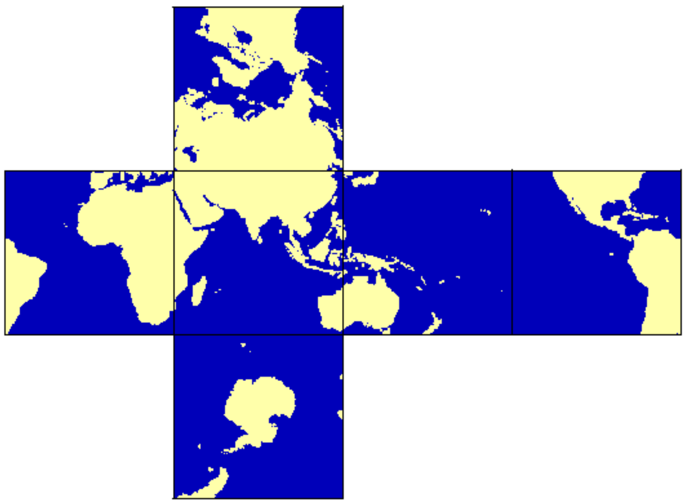


Cubed sphere: slightly non-orthogonal, good grid aspect ratio with 8 minor singularities (corners)



An equal-distance Gnomonic Cubed Sphere grid

- Defined by intersects of great circles with equal-distance along 12 edges
- Maximum local grid aspect ratio ~ 1.061
- Maximum global grid aspect ratio ~ 1.414



Commonly used resolution at GFDL:

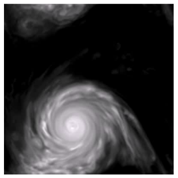
C2000, Δx = [3.92, 5.55] ~ 4.5 km

C360 , Δx ~ 25 km

C180 , Δx ~ 50 km

C90 , Δx ~ 100 km

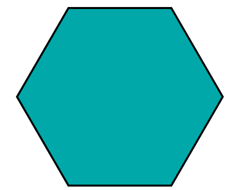
C48 , Δx ~ 200 km



Hurricane in a doubly periodic box

Can also be used as a regional model

Cubed-sphere *vs* icosahedral grid

		 + 
Grid uniformity/aspect ratio (time stepping efficiency)	~ 0.7 (Gnomonic grids)	0.7 ~ 0.8
Wave propagation	good	potentially better
Numerics: Advection & pressure gradients	High-order FV scheme developed & implemented	High-order FV scheme much more difficult to implement
Parallel efficiency	Potentially better	good
Grid nesting	Straightforward	difficult

Vertically Lagrangian non-hydrostatic FV core on the cubed sphere

$$\frac{\partial}{\partial t}\delta p^* + \nabla_h \cdot \left[\vec{V}\delta p^*\right] = 0$$

$$\frac{\partial}{\partial t}\left(\Theta\delta p^*\right) + \nabla_h \cdot \left[\vec{V}\Theta\delta p^*\right] = 0$$

$$\frac{\partial}{\partial t}u - \Omega\tilde{v}\sin\alpha + \frac{\partial}{\partial x}\left(\frac{\tilde{u}u+\tilde{v}v}{2}\right) = \boxed{\frac{\delta\Phi}{\delta p^*}\left[\frac{\partial p'}{\partial x}\right]_z} + \frac{\delta\Phi}{\delta\pi^*}\left[\frac{\partial \pi^*}{\partial x}\right]_z$$

$$\frac{\partial}{\partial t}v + \Omega\tilde{u}\sin\alpha + \frac{\partial}{\partial y}\left(\frac{\tilde{u}u+\tilde{v}v}{2}\right) = \boxed{\frac{\delta\Phi}{\delta p^*}\left[\frac{\partial p'}{\partial y}\right]_z} + \frac{\delta\Phi}{\delta\pi^*}\left[\frac{\partial \pi^*}{\partial y}\right]_z$$

$$\frac{\partial}{\partial t}\left(w\delta p^*\right) + \nabla_h \cdot \left[\vec{V}w\delta p^*\right] = g\delta p'$$

$$\delta m = \delta p^*/g = -\rho\delta Z$$

$$p = p^* + p'$$

$$\frac{\partial}{\partial t}\delta Z + \delta\left[\vec{V}\bullet\nabla_h Z\right] = \delta w$$

$$\xrightarrow{\textbf{\textit{hydrostatic}}}$$

$$\delta Z = \frac{-1}{g}C_p\Theta\delta\pi^*$$

Solved by either a Riemann Invariant Method (RIM) OR a Semi-Implicit Method (SIM)

Non-hydrostatic solvers

- **Semi Implicit Method (SIM)**
  - 4^{th} order finite-volume discretization
  - Stable for arbitrary time step size
  - Best for mid- to low resolution (100 km to 5 km)
- **Riemann Invariant Method (RIM)**
  - Based on conservation of Riemann invariants with open (for sound waves) top boundary condition
  - Fully explicit; conditionally stable (up to CFL ~ 100 in the PBL)
  - Suitable for cloud-resolving resolution (< 5 km)

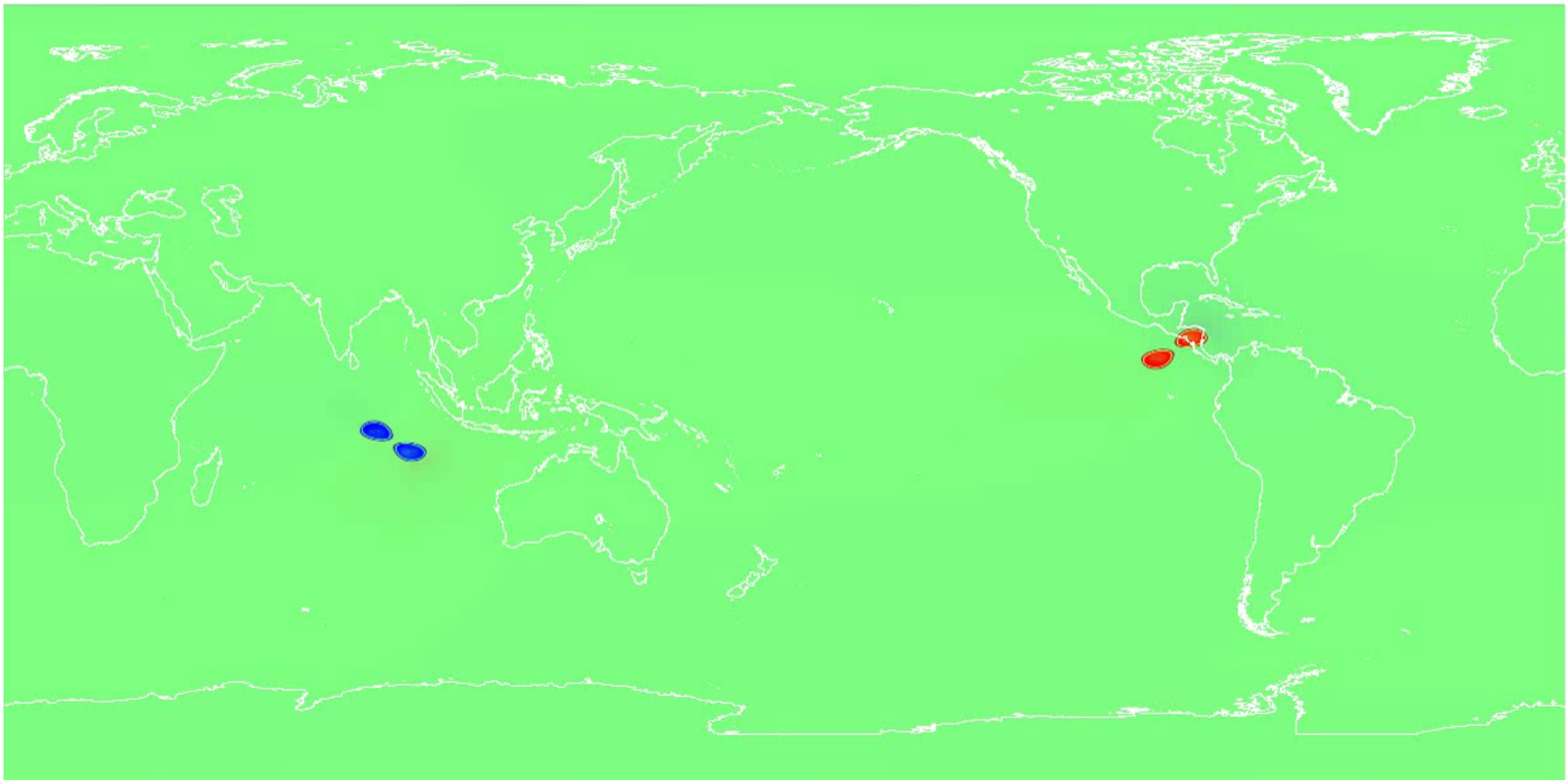
The finite-volume dynamical core enables the development at GFDL of a unified regional-global weather-climate modeling system suitable for all temporal-spatial scales

Examples:

- Simulations of TC climatology and response to warming scenarios (50km, Zhao et al. 2009)
- IPCC AR5 high-res time-slice versions (200, 50, and 25 *km*)
- TC seasonal predictions (25km; Chen and Lin 2011)
- Intra-seasonal TC predictions (25 *km, Gall et al 2010*)
- 5-10 day hurricane predictions (HFIP; 10 ~ 25 *km*)
- Global "cloud-resolving" experiments (3.5 km)
- Regional cloud-resolving radiative-convective equilibrium (1 km and 500m)

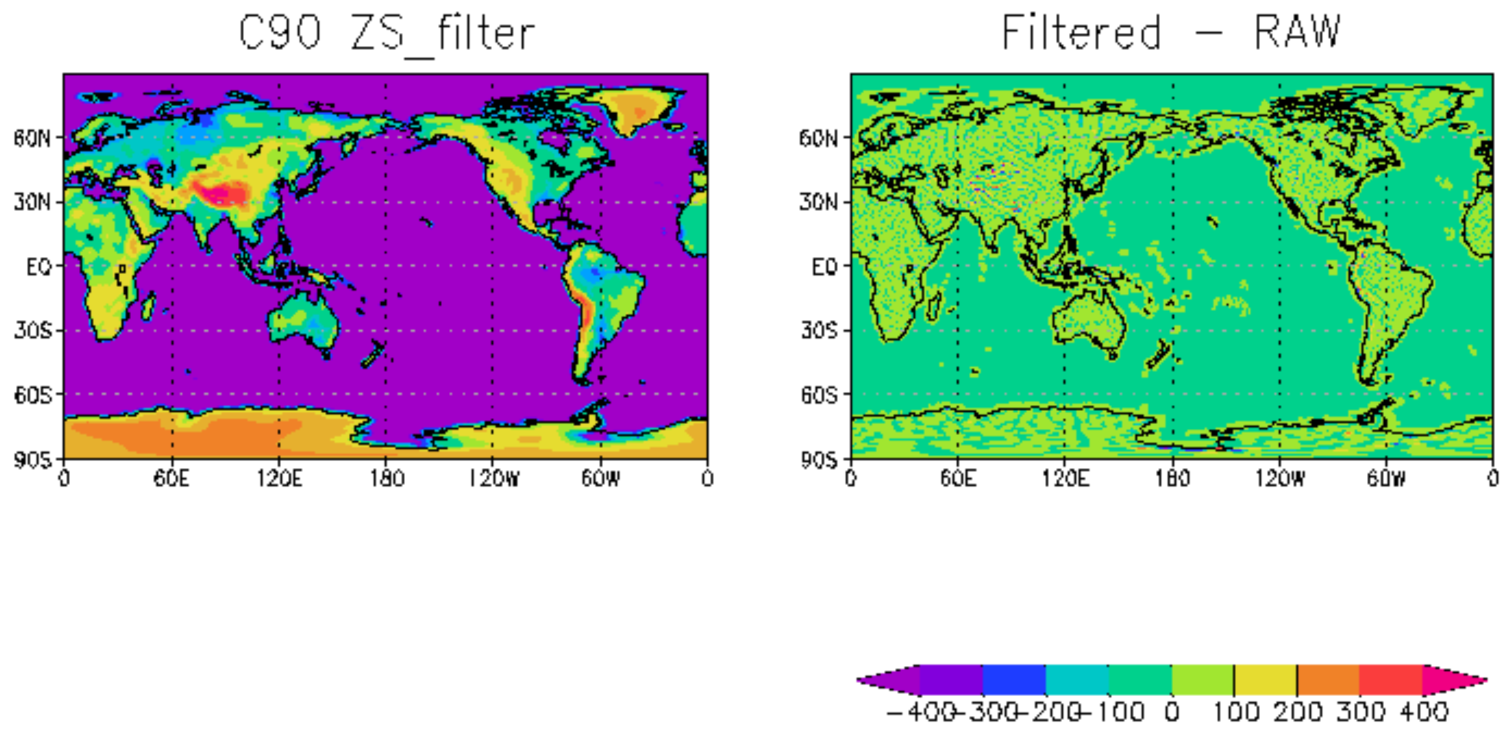
Nonlinear interaction of Rankine vortices

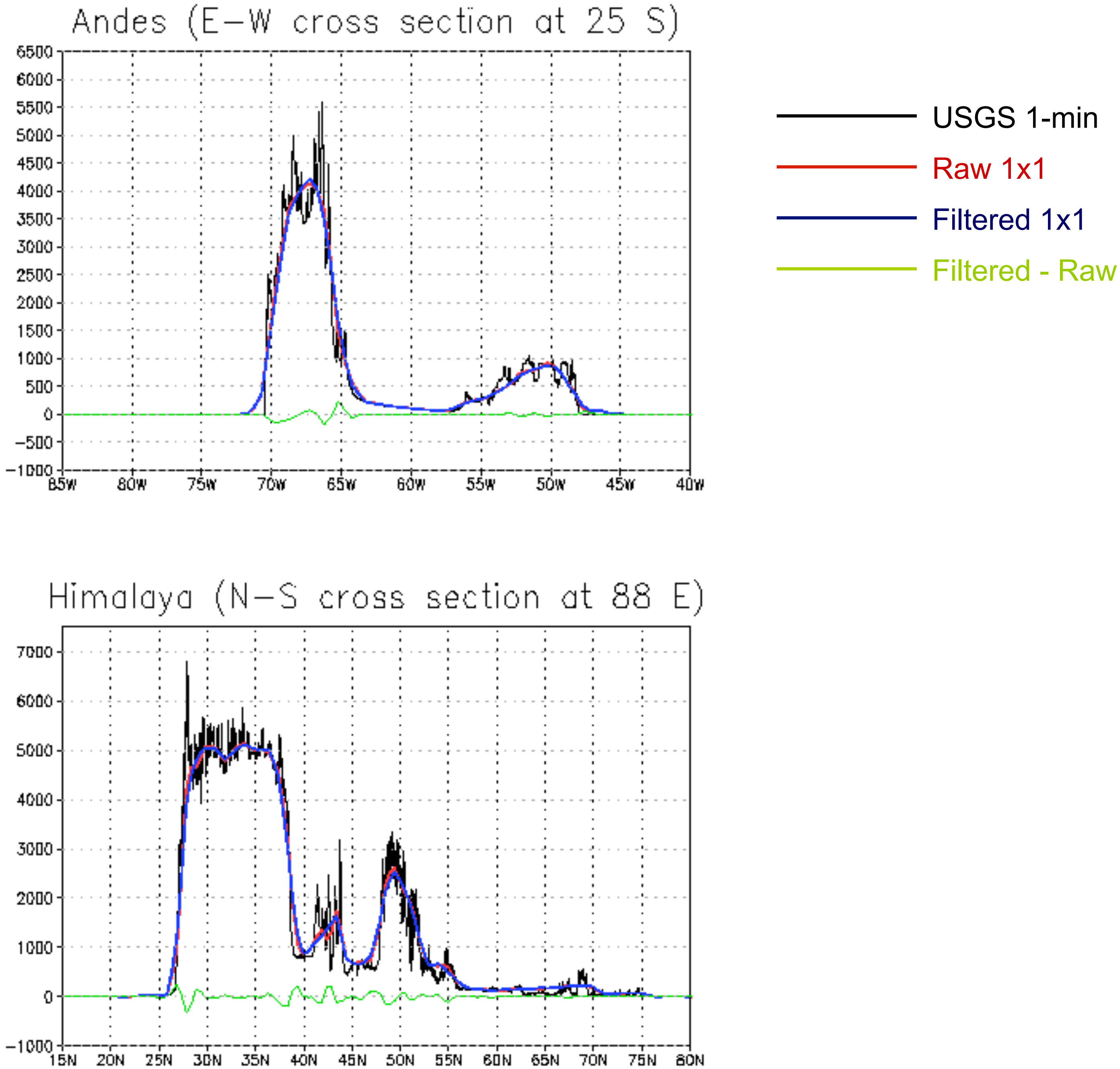
IC: Two pairs of anti-symmetrical Rankine vortex embeded in motionless “shallow water”



Island preserving terrain filter in GFDL cubed-sphere models

- Flux-form diffusive filter with fluxes set to zero if either side of the FV cell is covered completely by water
- Diffusive fluxes are designed to mainly filter out un-resolvable structures (2-delta-waves).




Andes (E–W cross section at 25 S)
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
–500
–1000
85W
80W
75W
70W
65W
60W
55W
50W
45W
40W
Himalaya (N–S cross section at 88 E)
7000
6000
5000
4000
3000
2000
1000
0
–1000
15N
20N
25N
30N
35N
40N
45N
50N
55N
60N
65N
70N
75N
80N
USGS 1-min
Raw 1x1
Filtered 1x1
Filtered - Raw

DJF precipitation in Western US:

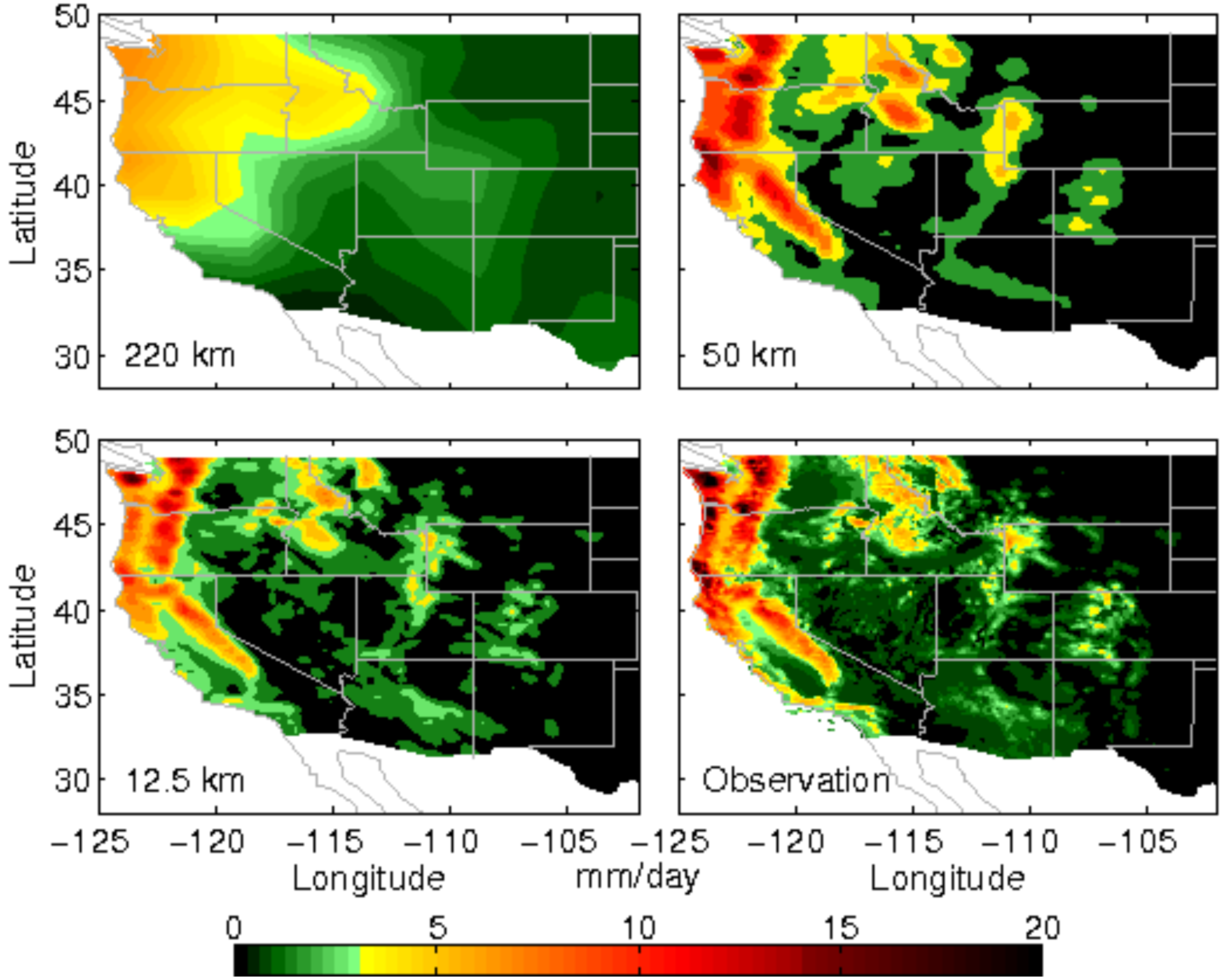
GFDL models vs. PRISM

GFDL AM2 (220 km)

50-km GFDL HiRAM

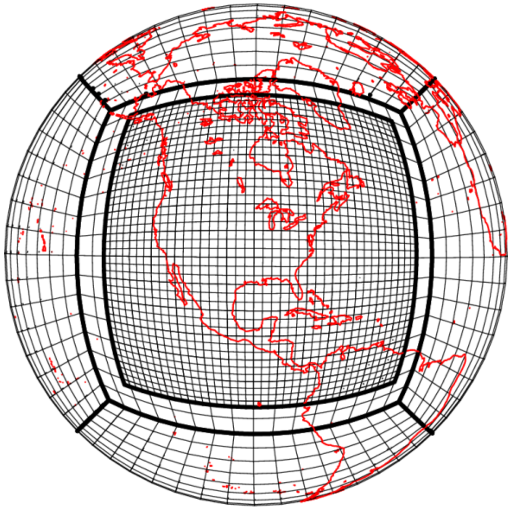
12.5-km GFDL HiRAM

PRISM (obs)



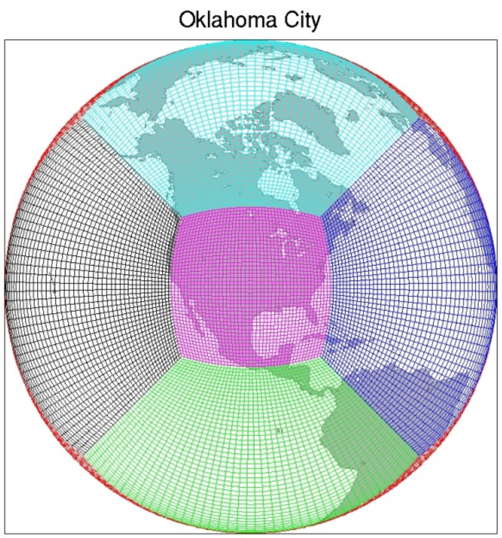
A. Nested regional-global climate model:

- 3X grid-size reduction; regional component can be run independently (for down-scaling) or coupled with global component to allow feedback to "global" changes

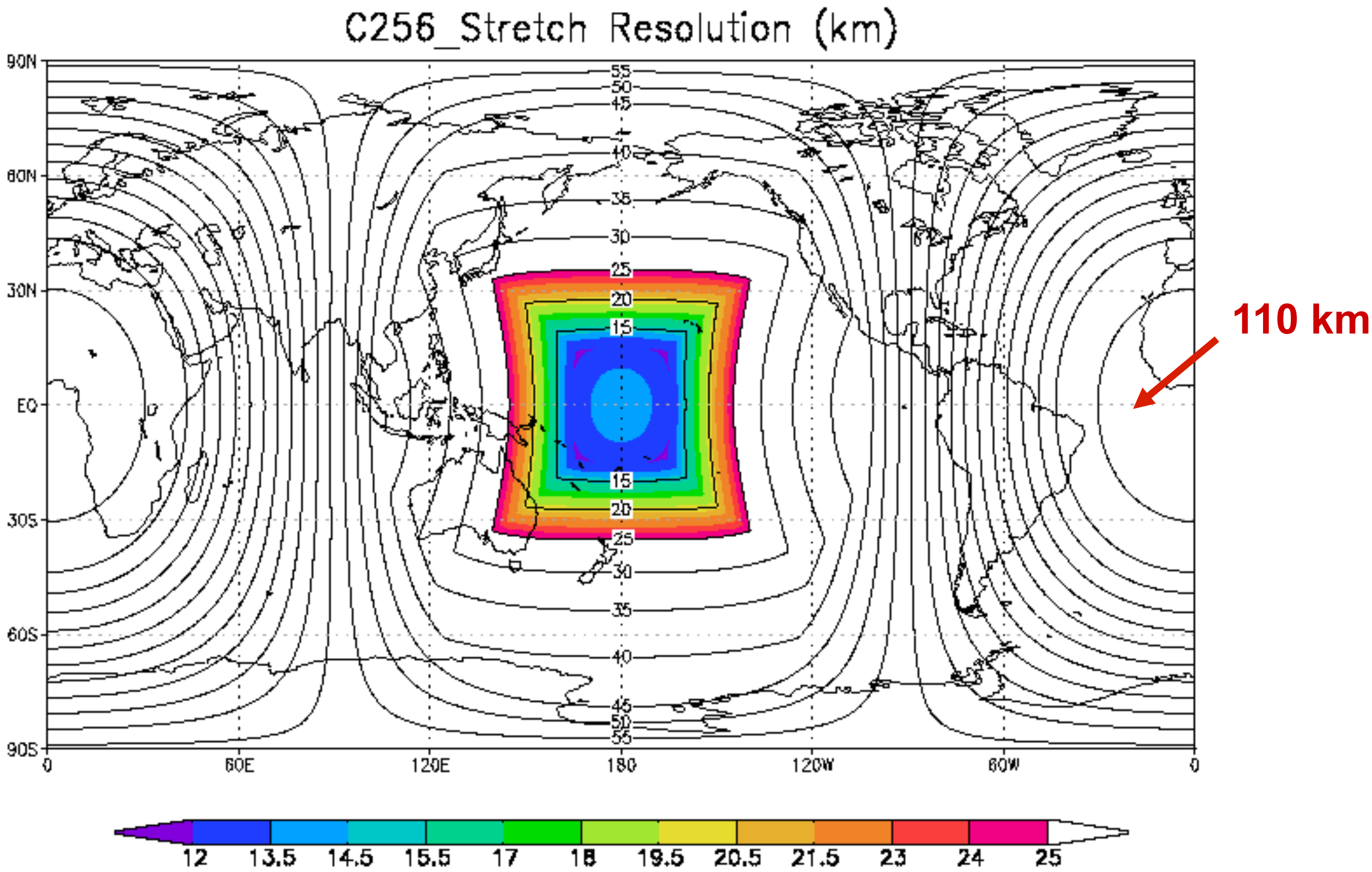


B. Variable resolution (via Schmidt transformation) climate model

- Single model framework with smooth transition in resolution with 3X grid-size reduction in target region (e.g., NA with ~ 4 km resolution); 3X enlargement on the back side

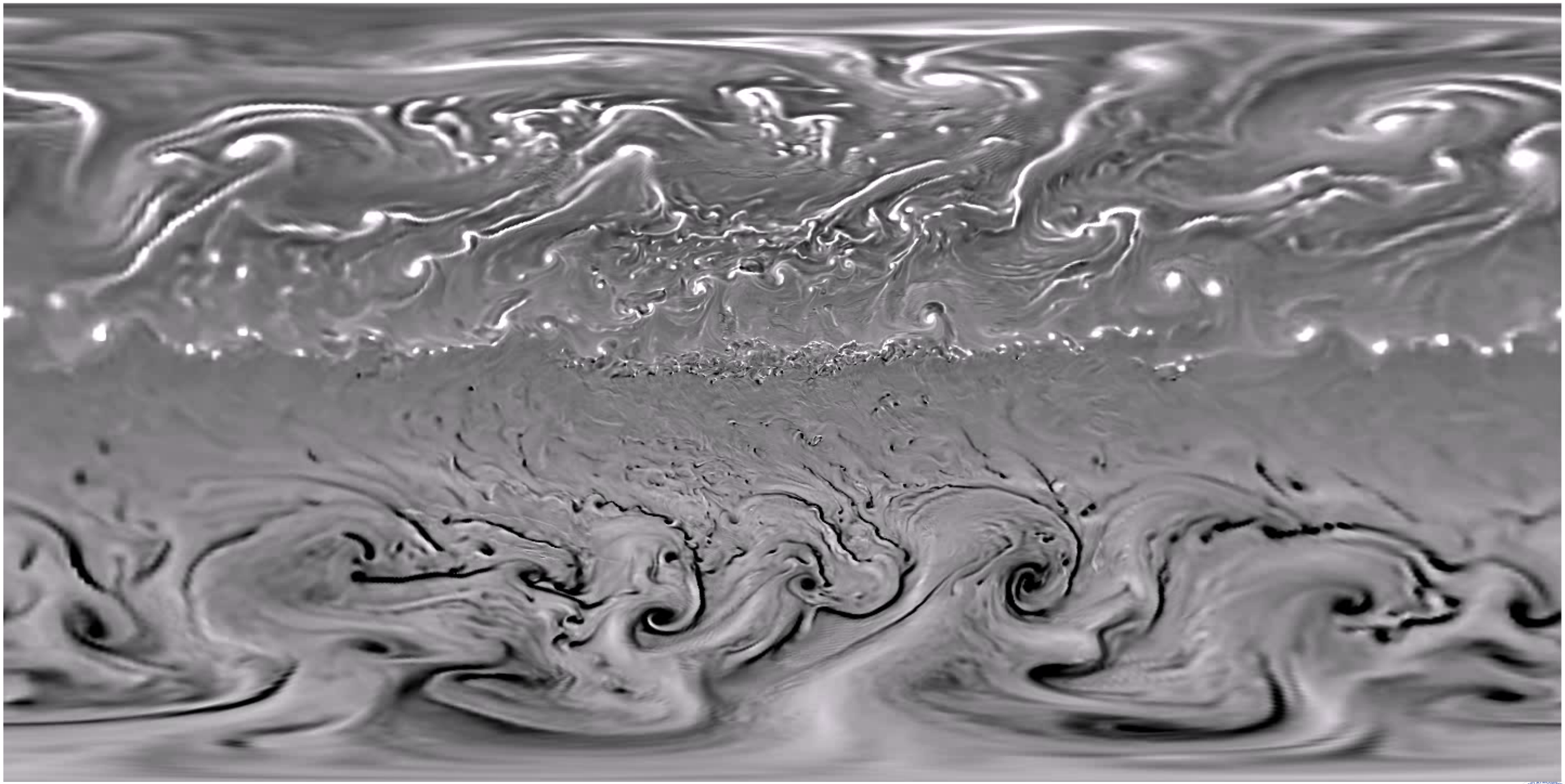


Aqua-planet test (HiRAM_MP)



Aqua-planet: surface vorticity

C256-Stretched (12 km – 110 km)

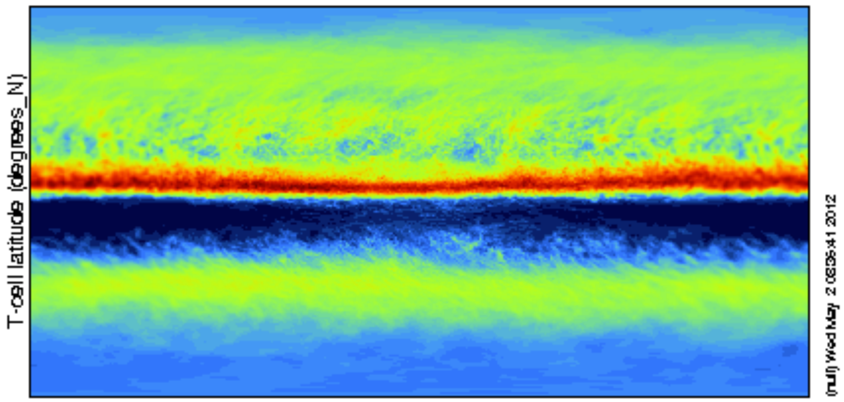


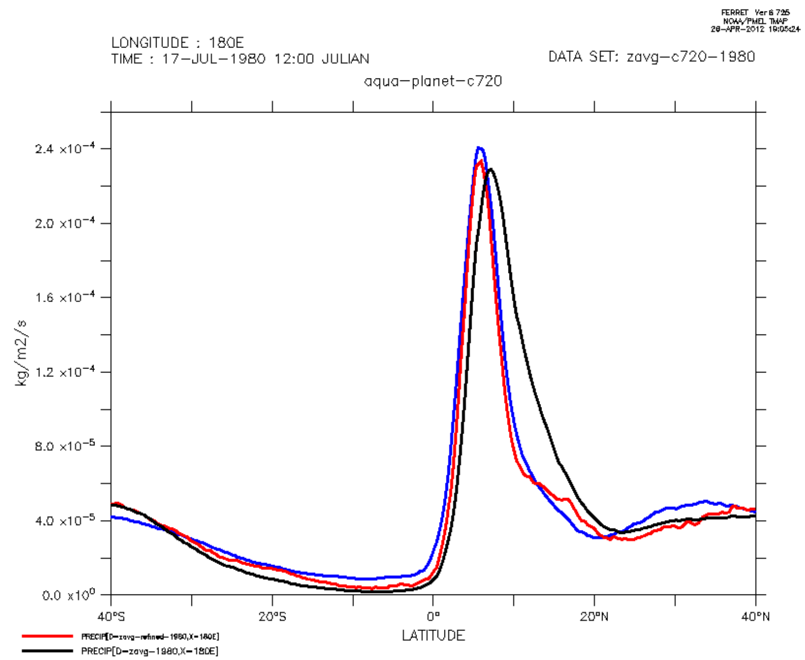
Aqua-planet with September SST

Variable resolution: 12 km – 110 km

Annual mean precipitation

total =3.5, LS=3.0 (mm/day)





Total precipitation rate (kg/m2/s)

Final notes:

Some unique properties of a consistently formulated finite-volume dynamical core

- With the vertically Lagrangian discretization, condensate loading effect is easy to implement, and there is no vertical CFL condition

- Dry air and tracer mass are conserved, and an initially constant tracer mixing ratio will remain constant

- In shallow water mode, PV is advected exactly the same as any other tracers. Since the advection is monotonic, no false PV gradient will be created in uniform PV region. In contrast, with a pure C grid formulation height field and absolute vorticity evolve differently, leading to inconsistent PV advection (the situation is much worse if time-splitting is used, e.g., some cloud resolving models based on C grid)

- On a non-rotating planet (or a Cartesian geometry), an initially irrotational flow (vorticity=0) will remain irrotational (*i.e.*, no false vorticity generation) with the FV core (not so with other models)